Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

Evanescence in Liquid Crystals

Elan Moritz [a]
[a] ELPA Research Group 34 Pomander Walk, Ridgewood, N.J., 07450
Version of record first published: 20 Apr 2011.

To cite this article: Elan Moritz (1977): Evanescence in Liquid Crystals, Molecular Crystals and Liquid Crystals, 41:3, 63-66

To link to this article: http://dx.doi.org/10.1080/01406567708071955

Mol. Cryst. Liq. Cryst., Vol. 41 (Letters), pp. 63-66

EVANESCENCE IN LIQUID CRYSTALS

ELAN MORITZ

ELPA Research Group
34 Pomander Walk, Ridgewood, N.J. 07450

(Received for Publication October 12, 1977)

ABSTRACT: The existence of a special type of electromagnetic oscillations in liquid crystals is brought out. The Saupe-Maier form of the Lorentz-Lorenz equation for nematics is used to show that there are four electromagnetic modes for each incident frequency as opposed to the commonly accepted two modes. For a nematic occupying the half space z>0 the results of an angular spectrum decomposition show that the phase velocity of the inhomogeneous evanescent waves are smaller than that of the associated homogeneous wave. Implications of the existence of evanescent waves in the smectic and cholesteric phases are discussed.

It is the purpose of this brief communication to alert investigators of the existence of a type of electromagnetic waves not commonly discussed, which have not heretofor been considered in the context of the liquid crystalline phases. These waves are the inhomogeneous or evanescent waves. In contrast with plane waves, these waves have surfaces of constant phase that do not necessarily coincide with the surfaces of constant amplitude. Furthermore, there is no net energy transfer via the evanescent mechanism.

To illustrate our discussion, consider the case of a nematic occupying the half space z>0. Without any rederivation, we may use the Saupe-Maier[1] form of the Lorentz-Lorenz[2] equations which follow the Neugebauer[3] field for a uniaxial crystal. The Lorentz-Lorenz equations then take the form:

$$(n_e^2-1)/(n_e^2+2) = \frac{4\pi}{3}\,[n/(A_1 + \alpha_{||}^{-1})] \qquad (1a)$$

$$(n_o^2-1)/(n_o^2+2) = \frac{4\pi}{3}\,[n/(A_2 + \alpha_{\perp}^{-1})] \qquad (1b)$$

where η_e and η_o are the indices of refraction of the extra-ordinary and ordinary waves, respectively, and where A_1 and A_2 $(A_1 + 2A_2 = 0)$ are parameters of the crystal structure The components of the polarizability tensor $\alpha_\perp$ and $\alpha_{||}$ are obtained from the full expression $\alpha_{\alpha\beta} = \alpha^o \delta_{\alpha\beta} + \alpha_\alpha S_{\alpha\beta}$ where $S_{\alpha\beta}$ is the order parameter.

We represent the incident electric and magnetic fields by:

$$\vec{E}(\vec{r},t) = \mathrm{Re}\{\vec{E}_o(\vec{r})e^{-iwt}\} \tag{2a}$$

$$\vec{H}(\vec{r},t) = \mathrm{Re}\{\vec{H}_o(\vec{r})e^{-iwt}\} \tag{2b}$$

and proceed with a self consistent analysis to obtain the field inside the nematic as a superposition of the incident field and the induced dipole field (special cases are discussed by Moritz and Franklin[4], references therein, and by Sengupta and Saupe[5]). Equations (1) along with the Ewald-Oseen extinction theorem[2] form the complete set of equations necessary for the solution of the molecular optics of a nematic for incident fields described by Equation (2).

Shewell and Wolf[6] have shown that the spatial part of the incident electric wave can be given by the angular spectrum composition:

$$\vec{E}_o(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \vec{F}(p,q)e^{i(px+qy+mz)}dpdq \tag{3}$$

with $\vec{F}(p,q)$ being the (complex) vector spectral amplitudes. When we set z=0 in Equation (3) and take the Fourier inverse, we find that the spectral amplitudes are naturally represented by the boundary values of the incident field in the z=0 plane. Thus Equation (3) represents a true mode expansion, where every plane wave is a solution of the Helmholtz equation for the medium. When $s^2 = p^2+q^2 \leq k^2$ we have homogeneous plane waves, while for $s^2 > k^2$ we have inhomogeneous or evanescent plane waves, where $k = w/c$ is the wave number of the incident field. The evanescent waves decay exponentially with increasing values of z.

To obtain the transmitted (refracted) ordinary and extra-ordinary waves in terms of the angular spectrum representation, let $p \to p' = p/\eta_o$ (p/η_e) and $q \to q' = q/\eta_o (q/\eta_e)$. The indices

η_o and η_e are obtained from the Saupe-Maier relation (1). The reflected wave exponents are obtained from the fact that the sum of fractions of transmitted and reflected amplitudes is unity.

We notice without proof that the reciprocal wavelength of the evanescent wave is $|s|$, and that there are four wavelengths of the fields inside the nematic (two directions each having evanescent and homogeneous components) and that for a given incident frequency w, the phase velocity of the evanescent wave is smaller than that of the homogeneous wave. In addition to all the above, there will be oscillations in the ratio of evanescent to homogeneous modes that are in direct correspondence with the order parameter tensor $S_{\alpha\beta}$.

If we have waves that are incident at an angle θ to the director $\hat{n}$, where ϕ_k is the azimuthal angle of the incident wave vector, we can obtain the condition for allowed modes:

$$s^2 = (\lambda n_1 + \sin\theta \cos\phi_k)^2 + (\lambda n_2)^2$$

the wavelength is related to the wave number by $\lambda = 2\pi/k$. With the aid of molecular structure relations it is then possible to obtain a variety of critical angles which will be described in a forthcoming paper.

We expect that it will be possible to detect the evanescent modes through shifts of light beams. The effect should be quite pronounced in the smectic and cholesteric phases due to multiple shifts due to a more constrained multilayer structure.[7]

REFERENCES

1. A. Saupe and G. Maier, Z. Naturforsch. 16, 816 (1961).

2. H.E.J. Neugebauer, Can. J. Phys. 18, 292 (1950).

3. M. Born and E. Wolf, Principles of Optics (Pergamon, Oxford and New York, 4th Ed., 1970).

4. E. Moritz and W. Franklin, Mol. Cryst. Liq. Cryst. 40, 229 (1977).

5. P. Sengupta and A. Saupe, Phys. Rev. A9, 2698 (1974).

6. J.R. Shewell and E. Wolf, J. Opt. Soc. 58, 1596 (1968).

7. O. Costa de Beauregard, C. Imbert and Y. Levy, Phys. Rev. D15, 3553 (1977).